4. GENERAL THEORY AND METHODOLOGY

4.1. INTRODUCTION

This part of the Polish National Report on Geodesy is the quadrennial report of works on theory, evaluation and methodology performed in Poland in a period from 1999 to 2002. It summarises investigations such as deformation analysis, least squares theory and evaluation, research on GPS, research in metrology, etc. Those activities were conducted mainly at the following research centres listed in an alphabetic order:

- Chair of Satellite Geodesy and Navigation, University of Warmia and Mazury in Olsztyn;
- Department of Geodesy and Geodynamics, Institute of Geodesy and Cartography in Warsaw;
- Department of Geodesy and Photogrammetry, Agricultural University in Wroclaw;
- Department of Mining Surveying and Environmental Engineering, University of Mining and Metallurgy in Cracow;
- Department of Planetary Geodesy, Space Research Centre, Polish Academy of Sciences in Warsaw;
- Institute of Geodesy, University of Warmia and Mazury in Olsztyn;
- Institute of Geodesy and Geodetic Astronomy, Warsaw University of Technology.

The content of the chapter is based on the material prepared by Marcin Barlik, Jozef Beluch, Stefan Cacon, Jan Krynski, Wojciech Pachelski, Zbigniew Wisniewski.

The bibliography of the related works is given in references.

Feasibility study on future targets and goals of geodesy as a discipline was conducted. In particular, the future activities in geodesy in 21st century at the Institute of Geodesy and Cartography, Warsaw were determined (Krynski and Sas-Uhrynowski, 2000).

Review of the first results of the CHAMP mission in terms of gravity field modelling, magnetic field modelling as well as ionosphere and troposphere investigations was conducted (Krynski, 2002).

4.2. DEFORMATION ANALYSIS

The definition of a geodetic reference frame by modern techniques (GPS) requires a determination of the systematic temporal changes of the defining stations' positions. Local and regional geodynamic studies need evaluation of inner (intraplate) velocities, reference points' velocities first of all. Relative velocities between selected EPN stations can be evaluated using different approaches (time series analysis, ITRF2000 velocities, NUVEL1A–NNR velocities APKIM velocities). The method of the mean trend congruency analysis of EPN stations coordinates time series (from weekly EUREF solutions) was proposed. The results of time series analysis of EPN stations, located no more then 700 km from local geodynamic network (LGN) in Sudety Mts. and Sudety Foreland (Central Europe), performed for the selection of the best reference stations are presented (Borkowski et al., 2001). Presented analysis has been limited only to the horizontal coordinates and velocities. The work was performed in the framework of Special Project of EUREF Permanent Network (http://www.epncb.oma.be/projects/sp_timeseries.html; Kenyeres et al., 2001).

The deformation analysis of upper lithosphere layer is of cognitive and practical significance. It is especially important for large urban and industry agglomeration areas and engineering object locations. An example of such analysis is the estimation of ground changes in Wroclaw on the basis of levelling conducted in 1968 and in 1998 (Cacon and Grzempowski, 2002). Reliability of geodetic deformation measurements was discussed by Cacon (2001).

Deformation measurements in the areas subdued to mining exploitation are substantial for the protection of the surface objects in such areas. Finding fixed reference points and estimating a realistic uncertainty of surveying are the major problems in deformation determination. Owing to the global satellite technology and the use of precise EDMs it becomes possible to achieve a sub-centimetre level of accuracy in the determination of the horizontal position and height of surveyed points. The results of survey and experiments concerning the measurement of deformation in the mining area of the salt mine in Wieliczka nearby Cracow (Krakow) were presented (Maciaszek and Szewczyk, 1999). They prove a usefulness of the combination of satellite and total station data to determine the post-exploitation deformations of the area.

Deviation decompositions between survey results concerning certain boundary processes in relation to the analytical description resulting from differential equations for regular and distributional solutions were investigated (Piwowarski and Bobula, 1999). Descriptive parameters of the discussed processes were analysed as certain statistical parameters estimated on the basis of survey results. Obtained solutions were verified for the purpose of an asymptotic post-mining dislocation process.

The problem of determination of one- and multi-stage prediction of post-mining surface dislocations was investigated (Piwowarski, 2000). The finite and chronologically ordered vector of surveyed data describes a variable herein. Completed surveys show that the analysed process can be presented as a composition of both deterministic process and a singular one. Hence the quantitative description of the kinetics of the process of forming dislocation has been assigned to the class of the stochastic model. The optimisation of one-stage prediction with use of the Durbin-Levinson algorithm has been carried out for a defined model of certain mining-geological conditions and surveying results. The obtained analytical representation and optimal prediction of the kinetics of vertical dislocations corresponded well to the surveying results that can be testified by adequate measures of the quality of description of the process.

The generalized kinematical model estimation parameters that can be the basis of engineering structures displacements and deformations description were investigated (Preweda, 2002). The possibility of implementation of the conditions that can guarantee deformations continuity, ability to transformation into dynamic model or simplification into commonly know models is a particular feature of the model. Parameters of the general linear model have been estimated by the least squares method, including Gauss-Markov conditions for a quadratic form described with Lagrange's function. Necessary conditions for minimum of Lagrange's function lead to the system of equations that has been solved and presented by means of block matrix generalized vertex.

4.3.LEAST SQUARES – THEORY AND EVALUATION

Some aspects of accuracy estimation of control networks in a form of polygonal traverses have been presented (Beluch, 1999a, 1999b). It concerns the densification of the control network using so called "free station" method. The formulae for mean square errors of

some functions of observations were developed. They can efficiently be used in planning observations considering the required accuracy in the projects.

Different correlation coefficients were analysed in terms of their application as well as their analytic and geometric interpretation (Czaja and Preweda, 2000).

Pearson correlation coefficient is always used for quantitative random variables since it is related with random variable distribution parameters and it has a straight geometric interpretation. The coefficient is determined by significance level for values predicted from linear regression model. Pearson correlation coefficients also define correlation matrix elements for a multidimensional random variable that is the basis of all statistic analyses of such variables.

A method for determination of the most effective, unbiased and invariant variance coefficient $\boldsymbol{\sigma}_0^2$ (in the model $\boldsymbol{\Sigma} = \boldsymbol{\sigma}_0^2\mathbf{Q}$ covariance matrix of observation results) in the class of quadratic estimators is given (Wisniewski, 1999a). The method follows from minimization of a function that determines the variance of the estimator as well as from application of vector transformations proposed in the paper (Wisniewski, 2002a). Basic relations between vectors derived from matrix elements are given in that paper. The transformations are carried out with matrices of transformation. The latter are defined taking advantage of developing operators. The task of those operators is to perform a unique transformation of a set of matrices into a block matrix. The vectors described, their transformations and the developing operators can provide a suitable tool for simplifying numerical operations as well as performing some theoretical analyses.

Robust method of estimation of variance coefficient $\boldsymbol{\sigma}_0^2$ has been developed (Wisniewski, 1999b). The concept of the method consists in searching for matrix $\boldsymbol{\Omega}_R$ that minimises variance of the quadratic form $\boldsymbol{\varepsilon}^T(\mathbf{R}^* \boldsymbol{\Omega}_R)\boldsymbol{\varepsilon}$ with $\mathbf{R} = \mathbf{R}(\boldsymbol{\varepsilon})$ being a function of observation errors $\boldsymbol{\varepsilon}$. Invariant, unbiased and most effective, with respect to the $\mathbf{R}$ matrix, estimator of variance coefficient is then $\boldsymbol{\sigma}_{0R}^2 = \boldsymbol{\varepsilon}^T\boldsymbol{\Omega}_R\boldsymbol{\varepsilon}$. The estimator obtained in such a way is the most effective for given matrices $\mathbf{R}_\gamma = \mathbf{R}_\gamma(\boldsymbol{\varepsilon})$ and $\boldsymbol{\gamma}$-matrix of observation error excess coefficients (Wisniewski, 2002b). Some forms of $\mathbf{R}_\gamma = \mathbf{R}_\gamma(\boldsymbol{\varepsilon})$ function and gain functions were derived (Wisniewski, 2002c).

The adjustment task that makes possible to estimate local coefficients of variance, asymmetry and kurtosis, assigned to respective observation group using the least-squares method is derived (Wisniewski and Kasietczuk, 1999). Next, using the maximum likelihood method the adjustment algorithm, making application of obtained estimators possible, was formulated.

4.4. RESEARCH ON GPS

In 1999 - 2002 the development of the original method and software for undifferenced processing GPS phase measurements was continued in the Space Research Centre of the Polish Academy of Sciences. It consisted of a programmed solution for GPS phase ambiguities and cycle slips in a single observation epoch. For that purpose a special package LAMBDA, kindly made available by the University of Delft, was implemented. The new version of the program PHASE was thoroughly tested. The obtained, tested and properly modified for a large number of observing stations and large inter-station distances, long observation sessions, and inhomogeneity of station receivers, features and properties of the PHASE software allow to be classified as a new technology available to relevant research projects. They can be particularly useful for a continuous service in real time of permanent observations for navigation and geodynamics, with a possible use of GPS, GNSS and Galileo systems.

The strategy for local precise GPS networks data processing allowing to connect the local networks with the global and the regional IGS/EPN networks was developed in (Bosy et al., 2003). In the research on precise local network data processing, the results of regional and global networks solutions (coordinates, systems of normal equations, tropospheric models and precise orbits) were used. High precision local network surveying requires taking into account the effects of ionospheric refraction when processing GPS data. This becomes particularly important during periods of high solar activity. It has been shown that the use of global and regional/local models of ionosphere improves the success rate of ambiguity resolution. Ionospheric activity modelling on a regional as well as local scale should provide a more accurate representation of the distribution of electrons. Those analyses have shown that local ionosphere models may be used for ambiguity resolution instead of global ones. A vital conclusion from this study is that the local model should be available with a significantly reduced delay time (just after the measurements) as opposed to 24 hours or more for global models. The presented strategy has been used for processing the latest observation campaigns (1997 – 2002) in the local GPS networks situated in test areas of the Sudety Mountains and the Fore-Sudetic Block.

Research on GPS positioning technique was continued at the Chair of Satellite Geodesy and Navigation, the University of Warmia and Mazury in Olsztyn (Baran and Oszczak, 2000). It concerned, in particular, the accuracy analysis of the rapid static technique (Bakula and Jarmolowski, 2001), accuracy of GPS kinematic positioning in real-time and in post-processing modes (Ciecko and Oszczak, 2001), accuracy of absolute GPS positioning with SA switched off (Oszczak et al., 2001).

4.5. RESEARCH ON GRAVITY

Results of research conducted at the Institute of Geodesy and Geodetic Astronomy of the Warsaw University of Technology on the application of the vertical gradient of gravity (Barlik, 2000d, 2000e; Zabek and Pachuta, 2000) and deflection of vertical (Barlik, 1999; 2000b) in the process of adjusting the geodetic control networks, gravimetric reductions (Pachuta et al., 1999; Pachuta, 2000), determination of local geoid etc. in (Barlik, 1999, 2000a, 2000c, 2000f; Walo, 2001), as well as concerning the application of the horizontal gravity gradient to determination of the geoid in (Barlik, 2001) were published.

4.6. RESEARCH IN METROLOGY

New methods in the field of surveying metrology were developed at the Metrological Laboratory of Department of Mining Surveying and Environmental Engineering of the University of Mining and Metallurgy in Cracow (Beluch et al., 2000). Conventional and electronic instruments as well as precise levelling staffs, both a traditional and bar-code ones were used in the research. A few models of bases for metrological tests of various surveying instruments, with particular emphasis on their precision, were analysed (Beluch, 2002). The usefulness of the model in determination of the EDM constants was evaluated. It was proved that the rms of the EDM constant could be reduced when two groups of sections: short and long ones were measured in test measurements. It was found that covariance of y co-ordinates of base points taken into account in the computation procedure makes the estimation of precision of constants determination more realistic.

The method of determination of linear expansion thermal coefficient for invar tape that is a part of precise levelling stuff was developed (Frukacz et al., 2000). The influence of rate temperature changes onto value of coefficient was determined. The way of the observations modification that eliminates imperfection of measurement series was shown.

Technology of comprehensive verification of precise levelling staffs based on the vertical calibrator with the use of HP 5529 laser interferometer was described (Pokrzywa et al., 2000).

References

Bakula M., Jarmolowski W., (2001): *Analiza dokladnosci wyznaczania wektorow GPS metoda Rapid Static,* Zeszyty Naukowe, Wyzsza Szkola Oficerska Sil Powietrznych, Deblin, Dodatek nr 2/2001, Materialy z konferencji naukowej "Satelitarne metody wyznaczania pozycji we wspolczesnej geodezji i nawigacji", Deblin, 22-23 May 2001, pp. 5-10.

Baran L.W., Oszczak S., (2000): *Zastosowanie techniki GPS w geodezji i geodynamice*, Nauka w Jubileuszu 50-lecia ART w Olsztynie, 24 June1999, Olsztyn, 2000, pp.103-115.

Barlik M., (1999): *Investigations of the geoid heights using vertical gradient anomalies near Grybow (Cracow District)*, Reports on Geodesy, No 4(45), 1999.

Barlik M., (2000a): *Gradientometric corrections to the geoidal heights levelling*, Reports on Geodesy IG&GA WUT, No 2(50), 2000.

Barlik M., (2000b): *On the determination of plumb line deflections using vertical gravity gradient anomalies*, Reports on Geodesy, IG&GA WUT, No 2(50), 2000.

Barlik M., (2000c): *On statistical prediction of the geoidal heights using vertical gravity gradient anomalies*, Reports on Geodesy, IG&GA WUT, No 2(50), 2000.

Barlik M., (2000d): *On the contribution of the vertical gravity gradient anomalies to the separation between geoid and Molodensky's quasigeoid (basing on the example of the geodetic test field near Grybow)*, Reports on Geodesy, IG&GA WUT, No 2(50), 2000.

Barlik M., (2000e): *Determination of the separation between geoid and quasigeoid using vertical gravity gradient observations along the gravimetric baseline in the meridian of astro-geodetic Observatory of Warsaw University of Technology at Jozefoslaw*, Reports on Geodesy, IG&GA WUT, No 2(50), 2000.

Barlik M., (2000f): *Gradientometric corrections to geoidal heights levelling on geodetic test field near Grybow (South Poland)*, XXV EGS General Assembly, Symposium G-12: Geodetic and geodynamic programmes of CEI (Central European Initiative), Nice, France, 2000, Reports on Geodesy, IG&GA WUT, No 2(50), 2000.

Barlik M., (2001): *Application of the horizontal gravity gradient anomalies in the geoid shape determination*, Reports on Geodesy, IG&GA WUT, No 1(56), 2001.

Beluch J., (1999a): *Problems of accuracy of control network in a form of polygonal traverses*, Scientific Meeting Celebrating the 80th Anniversary of the University Foundation, Cracow, pp. 115-127.

Beluch J., (1999b): *Accuracy aspect of the densification of geodetic network by mean of free stations*, Geodezja i Kartografia, Vol. XLVIII, No 1-2, pp. 19-37.

Beluch J., Mroz J., Pokrzywa A., Szczutko T., (2000): *Research works at the Metrological Laboratory at the Faculty of Mining Surveying and Environmental Engineering of the University of Mining and Metallurgy in Cracow*, Proceedings of the Scientific Conference Celebrating the 50th Anniversary of the Faculty of Mining Surveying and Environmental Engineering Foundation, Cracow, pp. 31–42.

Beluch J., (2002): *Evaluation of accuracy of various models of test bases and the method of determination of electronic distance meter constants*, Proceeding of the Geodesy and Environmental Engineering Commission of the Polish Academy of Sciences – Cracow, Geodesy, No 39, pp. 75-86.

Bosy J., Figurski M., Wielgosz P., (2003): *A strategy for GPS data processing in a precise local network during high solar activity*, GPS Solutions, Springer Verlag, 2003 (in press) (http://link.springer.de/link/service/journals/10291/tocs.htm).

Borkowski A., Bosy J., Kontny B., (2001): *The conditions of connection of local geodynamic network (LGN) with the EUREF permanent network (EPN)*, Proceedings of IAG 2001 Scientific Assembly 2–7 September 2001 – Budapest, Hungary (CD–ROM) (http://www.epncb.oma.be/papers.html).

Cacon S., (2001): *Problem of Reliability of Geodetic Measurements of Deformations of Engineering Object in Relation Object–Rock Mass*, Proceedings of the Institute of Geodesy and Cartography, Warsaw, t. XLVIII, z. 102, pp. 131–142 (in Polish).

Cacon S., Grzempowski P., (2002): *Effect of Ground Changes on the Reference Benchmarks in a Levelling Network of a Large City*, JS28 Integration of Techniques and Corrections to Achieve Accurate

Engineering Survey, XXII FIG International Congress ACSM/SPRS Annual Conference, 19-26 April 2002, Washington DC, USA (CD–ROM), online: http://www.fig.net/figtree/pub/fig_2002/Js28/JS28_cacon_grzempowski.pdf.

Ciecko A., Oszczak S., (2001): *Determination of Accuracy of GPS Kinematic Methods in Real-Time and Post-processing Modes,* Proceedings of IAG2001 Scientific Assembly "Vistas for Geodesy in the New Millenium", 2-8 September 2001, Budapest, Hungary, (CD).

Czaja J., Preweda E., (2001): *Statistic Analysis of Multidimensional Random variable considering Correlation and Prediction*, Geodezja, Vol.6, No 2, Wydawnictwo AGH, Krakow, pp. 129-145.

Frukacz M., Markiewicz M., Mroz J., Pokrzywa A., Szczutko T., (2000): *Determination of thermal expansibility coefficient of precise levelling rods in the Geodetic Comparator of University of Mining and Metallurgy in Cracow*, Wydawnictwo AGH, Geodezja, Vol. 6, No. 2, Krakow, pp. 91-111.

Kenyeres, A., Bosy J., Brockmann E., Bruyninx C., Caporali A., Hefty J., Jivall L., Kosters A., Poutanen M., Fernandes R., Stangl G., (2002): EPN *Special Project on "Time series analysis … " Preliminary Results and Future Prospects*, EUREF Publication No. 10, Mitteilungen des Bundesamtes für Kartographie und Geodäsie, Vol. 23, pp. 72–75, Frankfurt am Main, 2002.

Krynski J., Sas-Uhrynowski A., (2000): *Geodesy in 21st century at the Institute of Geodesy and Cartography, Warsaw* (in Polish), Proceedings of the Institute of Geodesy and Cartography, Special Issue 55 Aniversary of the Institute of Geodesy and Cartography, Warsaw, Poland, pp. 47-58.

Krynski J., (2002): *Misja satelitarna CHAMP – pierwsze wyniki i perspektywy badawcze*, Proceedings of the Institute of Geodesy and Cartography, Warsaw, t. XLIX, z. 105, pp. 7-47.

Maciaszek J., Szewczyk J., (1999): *The investigation of post-exploitation deformations of land surface using the GPS technique*, Das Markscheidewesen, Essen, No 2, pp. 245-249.

Oszczak S., Bakula M., Klecz A., Popielarczyk R., Gregorczyk R., (2001): *Dokladnosc metody absolutnej wyznaczania pozycji po wylaczeniu S/A,* Zeszyty Naukowe, Wyzsza Szkola Oficerska Sil Powietrznych, Deblin, Dodatek nr 2/2001, Materialy z konferencji naukowej "Satelitarne metody wyznaczania pozycji we wspolczesnej geodezji i nawigacji", Deblin, 22-23 May 2001, pp. 96-101.

Pachuta A., (2000): *Real time determination of the geodetic co-ordinates reductions in the Earth's gravity field*, Reports on Geodesy, IG&GA WUT, No 6(52), 2000.

Pachuta A., Barlik M., Kalinowska-Sledzinska B., Walo J., (1999): *Automation of the determination of vertical gravity gradient*, Reports on Geodesy, No 4(45), 1999.

Piwowarski W., Bobula E., (1999): *Problems resulting from global analysis of the asymptotic description of vertical dislocations*. Scientific Meeting Celebrating the 80th Anniversary of the University Foundation, Cracow, pp. 199-210.

Piwowarski W., (2000): *Mathematical modelling of post-mining dislocations kinetics as a quasi-random process*, Geodezja i Kartografia, Vol. XLIX, No 1, Warszawa, pp. 21-38.

Pokrzywa A., Mroz J., Szczutko T., Ruchel J., Przewiezlikowska A., (2000): *Technology of comprehensive verification of precise levelling staffs with the use of the HP 5529A laser interferometer*, Wydawnictwo AGH, Geodezja, Vol.6, No 1, Krakow, pp. 11-17.

Preweda E., (2002): *Estimation of Parameters of Deformations Kinematics Model*, Wydawnictwa AGH, Dissertations Monographies, No 110, Krakow, pp. 7-97.

Walo J., (2001): *How to get a precise local levelling geoid*, Reports on Geodesy, IG&GA WUT, No 1(56), 2001.

Wisniewski Z., (1999a): *The most effective estimator of variance coefficient in geodetic observation systems*, Geodezja i Kartografia, t. XLVIII, z. 3-4, 1999, pp. 105-131.

Wisniewski Z., (1999b): *A concept of robust estimation of variance coefficient (VR-estimation)*, Bolletino di Geodesia e Scienze Affini, No 3, 1999, pp. 291-310.

Wisniewski Z., Kasietczuk B., (1999): *Estimation of some parameters of probability distribution for measurement errors in nonhomogenous sets of observations*, Proceedings of The 9th FIG Symposium on Deformation Measurements, 1999, Olsztyn, pp. 267-276.

Wisniewski Z., (2002a): *Vectors derived from matrix elements and their basic transformations*, Technical Sciences, No 5, 2002, pp. 121-143.

Wisniewski Z., (2002b): *Robust estimation of variance coefficient with excess of observation errors distribution (VR_γ - estimation). Part I. Theory*, Technical Sciences, No 5, 2002, pp. 73-92.

Wisniewski Z., (2002c): *Robust estimation of variance coefficient with excess of observation errors distribution (VR_γ - estimation). Part II. Analysis*, Technical Sciences, No 5, 2002, pp. 93-119.

Wisniewski Z., Czaplewski K., (2002): *The proposed method of expanding the navigation system during the near-shore navigation*, Proceedings of III International Scientific and Technical Conference "The Part of Navigation in Support of Human Activity on the Sea", Naval University of Gdynia, 2002, pp. 461-471.

Zabek Z., Pachuta A., (2000): *Absolute measurements with Polish ballistic gravimeter*, Workshop on High Precision Gravity Measurements with Application to Geodynamics and Second GGP Workshop, Walferdange, Luxembourg, 24-26 March 1999, Ed. by B. Ducarme & J. Barthelemy, Luxembourg, 2000.